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The Effects of Illuminants and Standard Observers Combination on Relationship between Spectrophotometric Error and Colorimetric Inaccuracy

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ABSTRACT

The colorimetric error depends on the spectrophotometric inaccuracy. In this paper, a new method is introduced for determining the relationship between spectrophotometric error and colorimetric inaccuracy. The error propagation in colorimetric parameter calculation is evaluated using a linear relation between variance of reflectance spectra and CIE tristimulus values. This linear formula calculates the variance of CIE tristimulus values from the variance of reflectance spectra with sufficient accuracy. In addition, the effect of illuminants and standard colorimetric observers on the error propagation from spectral data into CIE tristimulus values is studied using the proposed relationship. The results indicate that the effect of illuminants is more than the effect of standard observer. In addition, the Standard deviation of CIE tristimulus values depends on their respective coefficients. Prog. Color Colorants Coat. 4(2011), 59-69. © Institute for Color Science and Technology.

1. Introduction

Appearance is a feature of visual experience by which matters are recognized. The color of a colored object depends on three factors, the nature of the illumination under which the colored object is viewed, the interaction of the illuminating radiation with the colored object in the surface layers, mainly within the visible section of the electromagnetic spectrum and the performance of the radiation that is reflected or transmitted from the colored object to make the sensation of color in the human eye system. Appearance changes considerably when these factors are changed. The appearance of the object can be defined by color and reflectance (absorbance) spectrum. The object reflectance determines what proportion of the light incident on the surface is reflected, which is a function of wavelength. The color appearance of the object will therefore depend on the product of the reflectance of the object, spectral power distribution of illuminants and color matching function of standard colorimetric observers [1-4]. CIE *XYZ* tristimulus values can be calculated by summation of the reflectance (R_λ), the relative spectral energy distributions of the illuminant (E_λ), and the standard observer functions $(\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda)$ as follows:

$$X = K \times \sum_{\lambda} E_\lambda \times R_\lambda \times \bar{x}_\lambda \quad (1)$$

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$$Y = K \times \sum_{\lambda} E_{\lambda} \times R_{\lambda} \times \bar{y}_{\lambda} \quad (2)$$

$$Z = K \times \sum_{\lambda} E_{\lambda} \times R_{\lambda} \times \bar{z}_{\lambda} \quad (3)$$

where $K = \frac{100}{\sum_{\lambda} E_{\lambda} \times \bar{y}_{\lambda}}$ and λ is wavelength in the visible range of electromagnetic radiation.

The statistical techniques can be used to model spectrophotomeric errors and measure their effect on colorimetric inaccuracy. During 1980s, a statistical technique was suggested by Robertson [5] and Berns and Petersen [6], which was very successful in diagnosing spectrophotometric errors and correcting them in order to improve accuracy. The technique is based on the measurement and analysis of a set of achromatic and chromatic calibrated standard reference materials. The method can be used to measure the accuracy and reproducibility of spectrophotometric measurements [5-6].

In every physical measure, it is important to report the maximum amount by which any measured value might be in error. The error propagation techniques can be used to determine the uncertainty in a calculated value due to random errors in its constituents. Fairchild et al. [7] studied the propagation of random errors in spectrophotometric colorimetry. They obtained that the standard deviations of CIELAB coordinates for typical measurements can be as high as 0.258 due to random errors in the calibration chain [7]. Burns et al. [8], studied the error propagation analysis in color measurement and imaging. They applied multivariate error-propagation analysis to color-signal transformations. The results of their study indicate how linear, matrix, and nonlinear transformations influence the mean, variance, and covariance of color-measurements. Some expressions were suggested that evaluate error propagation for a spectrophotometer, colorimeter system. For example, error propagation from tristimulus values to CIELAB coordinates [8]. Burns et al. [9], suggested an abridged technique to identify spectrophotometeric errors. The error simulation indicates that the lack of spectrophotometeric accuracy leads to colorimetric inaccuracy. The scale of the inaccuracy depends on the particular error and the spectral nature of the samples used to represent the errors [9]. Gardner [10] described the uncertainty propagation through fitting and inverse spectral measurements. The results of their study show that interpolation is best technique for the interpolation of NIST-provided spectral irradiance values rather than fitting technique [10]. Early and Nadal [11], offered a systematic, analytical approach to uncertainty analysis, which conforms to currently accepted practice for reflectance colorimetry. They suggested an equation for describing the relationship between signals and the spectral reflectance factor, and correlations both between signals at the same wavelength and between reflectance values at different wavelengths. The uncertainties propagation depends on the reflectance spectra form of samples, so that the largest uncertainties are for the red specimen, whose reflectance increases rapidly with wavelength. [11].

1.1. The variance of function:

The standard deviation and variance is related sign of variation of data within a population or sample. The standard deviation and variance of a variable is a measure of statistical dispersion, averaging the squared distance of its possible value from the expected value as mean value. The sample standard deviation is the square root of the sample variance. The variance is usually denoted σ^2, which is defined by equation 4.

$$\sigma^2 = E(X - \mu)^2 \quad (4)$$

Variance is a positive value, because the squares are positive or zero. If all values are multiplied by a constant value, the variance is scaled by the square of that constant value. The variance of a finite summation of uncorrelated random variables is equal to the summation of their variances. Assume that the variable can be partitioned into subgroups and sub-variables according to some second variable. Then the variance of the total is equal to the mean of the variances of the subgroups plus the variance of the means of the subgroups. This property is known as variance decomposition or the law of total variance and plays an important role in the analysis of variance. In a more general case, if the subgroups have not the same sizes, then they should be weighted proportionally to their dimension in the calculation of the means and variances [12-15].

In the delta method, the second-order expansions of Taylor are used to approximate the variance of a function of one or more random independent variables (equation 5).

$$\mathrm{VAR}[f(X)] \approx (f'(E[X]))^2 \times \mathrm{Var}[X] \quad (5)$$

where f' is first derivative of f and $\mathrm{Var}[X]$ denote variance of X.

Suppose we are interested in a quantity W, whose values can be found when values of several independent random variables $x_1, x_2, x_3, \ldots\ldots x_k$ are known, i.e. W is a function of these variables:

$$W = f(x_1, x_2, x_3, \ldots, x_k) \quad (6)$$

Now assume an experiment consists of measuring each the x's and calculating the value of W. If this experiment is repeated many times, a sequence of W values will be generated, which, in general, will vary in each report from repeat to repeat and will have a probability distribution. This distribution is called the sampling distribution of W, and the general difficult problem is to calculate the form of the sampling distribution when the distribution of the x's are given. However, the mean μ_W and the variance σ_W^2 of the W can easily be found. Suppose the distributions of $x_1, x_2, x_3, \ldots\ldots x_k$ have mean $\mu_1, \mu_2, \mu_3, \ldots\ldots \mu_k$ and variances of $\sigma_{x_1}^2, \sigma_{x_2}^2, \sigma_{x_3}^2, \ldots\ldots \sigma_{x_k}^2$. Then, it can be shown that:

$$\mu_W \approx f(\mu_1, \mu_2, \mu_3, \ldots, \mu_k) \quad (7)$$

and for variance σ_W^2:

$$\sigma_W^2 \approx \left(\frac{\partial w}{\partial x_1}\right)^2 \times \sigma_1^2 + \left(\frac{\partial w}{\partial x_2}\right)^2 \times \sigma_2^2 + \left(\frac{\partial w}{\partial x_3}\right)^2 \times \sigma_3^2 + \ldots\ldots + \left(\frac{\partial w}{\partial x_k}\right)^2 \times \sigma_k^2 \quad (8)$$

where the partial derivatives are evaluated at $x_1 = \mu_1, x_2 = \mu_2, x_3 = \mu_3, \ldots, x_k = \mu_k$. These equations are exactly true when the function 6 is linear as equation 9:

$$W = a_1 \times x_1 + a_2 \times x_2 + a_3 \times x_3 + \ldots\ldots + a_k \times x_k \quad (9)$$

The variance of the W function is calculated by equation 10:

$$\sigma_w^2 = (a_1)^2 \times \sigma_{X_1}^2 + (a_2)^2 \times \sigma_{X_2}^2 + \ldots\ldots + (a_K)^2 \times \sigma_{X_K}^2 \quad (10)$$

where the a_i (i=1, 2…, k) are constants [15-18].

This work explains a numerical analysis of spectrophotometric errors effects on colorimetric error. In this regard, the effect of illuminants and observers combination on the transportation of error from reflectance data to CIE tristimulus values was evaluated. Also, the sensitivity coefficients for CIEXYZ color parameter values were calculated.

2. Experimental

2.1. Materials and methods

The spectral reflectance data of 1269 Munsell color chips were available from the web site of university of Joensuu [19]. The CIE tristimulus values of Munsell color chips are calculated under various illuminants (such CIE A, CIE D65 and EE (equal energy)) and 2 and 10-degree standards colorimetric observer. The spectral power distribution of CIE A, CIE D65 and EE illuminants are shown in Figure 1.

3. Results and discussion

For evaluating the effect of illuminants and observers combination on the transportation of error and variation from reflectance data to CIE tristimulus values, the standard deviation of reflectance spectra of Munsell color chips were calculated at 5 nm intervals from 400 to 700 nm.

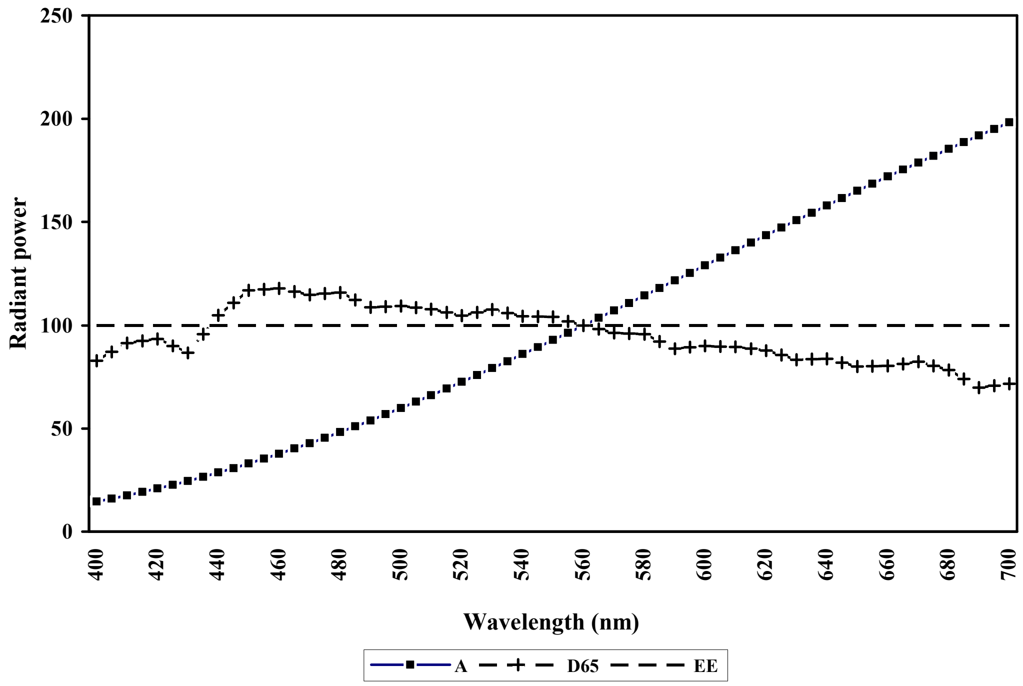

Figure 1: The spectral power distribution of CIE A, CIE D65 and EE illuminants.

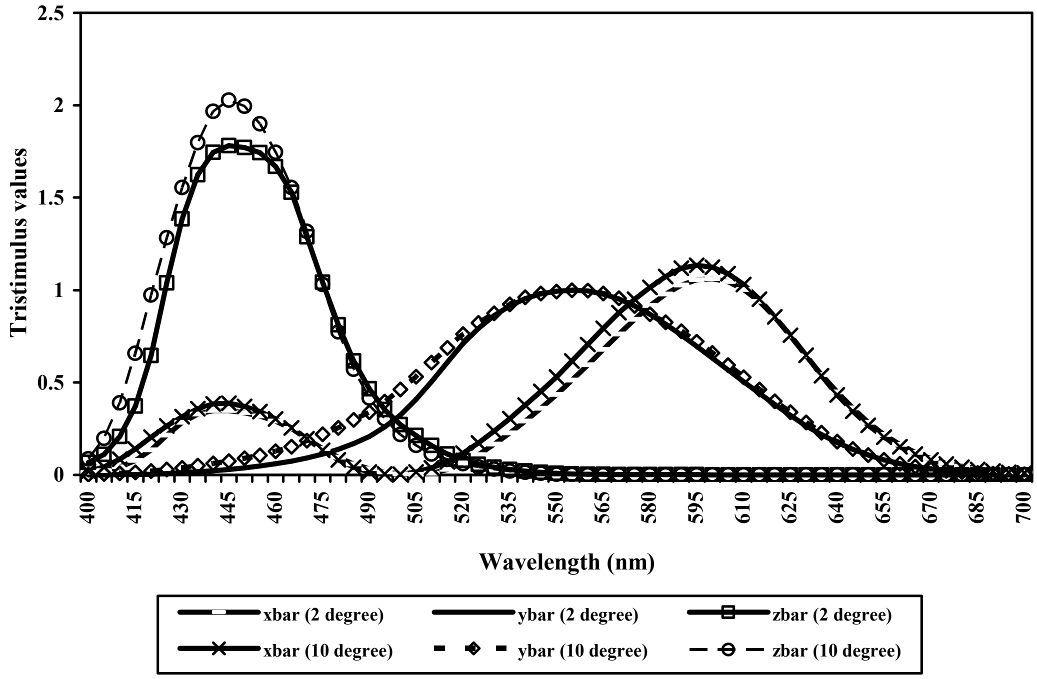

Figure 2: Color matching functions of 2 and 10 -degree standard observers.

The standards deviation of reflectance spectra for each wavelength for the set of patch on a Munsell hue plane are shown in Tables 1 and 2.

Table 1: Standard deviation of reflectance spectra of Munsell color chips (400-550 nm).

Wavelength (nm)	Munsell Hue									
	R	YR	Y	GY	G	BG	B	PB	P	RP
400	0.113	0.111	0.106	0.105	0.105	0.112	0.116	0.114	0.111	0.112
405	0.138	0.134	0.124	0.125	0.128	0.140	0.150	0.151	0.147	0.144
410	0.153	0.145	0.133	0.135	0.142	0.157	0.173	0.175	0.171	0.164
415	0.159	0.148	0.136	0.139	0.148	0.166	0.184	0.189	0.183	0.172
420	0.160	0.149	0.136	0.140	0.150	0.169	0.189	0.195	0.188	0.174
425	0.159	0.148	0.136	0.140	0.151	0.170	0.191	0.197	0.189	0.174
430	0.159	0.148	0.136	0.141	0.151	0.172	0.192	0.198	0.190	0.174
435	0.158	0.148	0.137	0.141	0.152	0.173	0.193	0.199	0.190	0.174
440	0.158	0.148	0.137	0.142	0.153	0.174	0.195	0.200	0.190	0.174
445	0.157	0.147	0.138	0.143	0.154	0.175	0.196	0.200	0.189	0.173
450	0.156	0.147	0.138	0.143	0.154	0.176	0.197	0.200	0.189	0.172
455	0.156	0.148	0.139	0.145	0.156	0.178	0.197	0.200	0.189	0.172
460	0.157	0.148	0.141	0.148	0.158	0.180	0.198	0.200	0.188	0.172
465	0.157	0.148	0.143	0.154	0.161	0.183	0.198	0.199	0.187	0.173
470	0.158	0.148	0.145	0.161	0.165	0.186	0.199	0.198	0.187	0.173
475	0.158	0.148	0.148	0.170	0.169	0.189	0.198	0.196	0.185	0.173
480	0.157	0.147	0.152	0.179	0.173	0.191	0.198	0.194	0.184	0.171
485	0.154	0.146	0.157	0.188	0.177	0.193	0.197	0.192	0.181	0.169
490	0.152	0.145	0.162	0.196	0.180	0.194	0.197	0.190	0.180	0.167
495	0.151	0.144	0.169	0.204	0.185	0.195	0.196	0.188	0.178	0.166
500	0.152	0.146	0.175	0.209	0.188	0.196	0.196	0.186	0.177	0.166
505	0.154	0.150	0.182	0.212	0.190	0.196	0.195	0.184	0.176	0.166
510	0.155	0.154	0.185	0.212	0.191	0.195	0.194	0.182	0.174	0.166
515	0.155	0.159	0.187	0.209	0.191	0.194	0.192	0.179	0.171	0.165
520	0.152	0.162	0.189	0.207	0.191	0.194	0.191	0.176	0.169	0.162
525	0.150	0.162	0.192	0.205	0.189	0.192	0.189	0.174	0.166	0.159
530	0.148	0.162	0.197	0.204	0.189	0.191	0.187	0.173	0.165	0.158
535	0.147	0.163	0.202	0.204	0.188	0.190	0.185	0.172	0.164	0.158
540	0.149	0.166	0.206	0.204	0.186	0.188	0.183	0.171	0.164	0.159
545	0.155	0.171	0.209	0.203	0.184	0.187	0.181	0.170	0.165	0.163
550	0.164	0.178	0.212	0.203	0.183	0.185	0.178	0.169	0.166	0.168

The standard deviations of the CIE tristimulus values across the set of patches on a Munsell hue plane are calculated under various illuminants (such CIE A, CIE D65 and EE) and 2 and 10-degree standard colorimetric observers. The standards deviation of X, Y and Z tristimulus values under CIE A, CIE D65 and EE illuminants and 10 degree standard colorimetric observers in relation to Munsell constant-hue plane are shown in Tables 3 and 4, respectively. As shown in these tables, the standard deviation of X, Y and Z tristimulus values depends on the standard illuminants and standard colorimetric observers. The standard deviation of X tristimulus value under CIE A illuminants is more than CIE D65 and EE illuminants. Lower standard deviation of X tristimulus value is obtained under CIE D65 illuminants. The standard deviation of Y tristimulus value under CIE A illuminant is the highest. The standard deviation of Z tristimulus value under CIE.

Table 2. Standard deviation of reflectance spectra of Munsell color chips (555-700 nm).

Wavelength (nm)	Munsell Hue									
	R	YR	Y	GY	G	BG	B	PB	P	RP
555	0.174	0.187	0.214	0.203	0.181	0.182	0.175	0.166	0.166	0.172
560	0.184	0.195	0.216	0.203	0.179	0.181	0.174	0.166	0.166	0.175
565	0.193	0.200	0.216	0.202	0.177	0.178	0.170	0.163	0.167	0.177
570	0.198	0.204	0.216	0.201	0.174	0.175	0.168	0.163	0.168	0.180
575	0.201	0.206	0.217	0.201	0.172	0.172	0.166	0.163	0.170	0.184
580	0.203	0.208	0.217	0.200	0.169	0.169	0.165	0.164	0.173	0.189
585	0.204	0.210	0.217	0.199	0.166	0.165	0.163	0.164	0.176	0.193
590	0.205	0.211	0.218	0.198	0.162	0.161	0.162	0.165	0.180	0.197
595	0.206	0.213	0.218	0.196	0.158	0.157	0.160	0.166	0.183	0.200
600	0.207	0.214	0.218	0.195	0.154	0.153	0.159	0.166	0.184	0.203
605	0.207	0.214	0.218	0.194	0.149	0.150	0.157	0.165	0.186	0.205
610	0.209	0.215	0.218	0.193	0.146	0.148	0.156	0.165	0.187	0.206
615	0.210	0.216	0.219	0.191	0.142	0.146	0.156	0.164	0.188	0.208
620	0.211	0.216	0.219	0.190	0.139	0.145	0.156	0.164	0.189	0.209
625	0.213	0.217	0.219	0.189	0.137	0.144	0.156	0.164	0.190	0.209
630	0.215	0.217	0.220	0.189	0.136	0.144	0.156	0.165	0.191	0.211
635	0.216	0.218	0.220	0.188	0.134	0.143	0.157	0.167	0.191	0.211
640	0.217	0.218	0.220	0.188	0.132	0.142	0.157	0.169	0.192	0.212
645	0.218	0.218	0.221	0.187	0.131	0.142	0.157	0.171	0.192	0.213
650	0.219	0.219	0.221	0.186	0.129	0.141	0.158	0.173	0.193	0.213
655	0.219	0.219	0.221	0.186	0.128	0.142	0.158	0.175	0.193	0.214
660	0.220	0.219	0.222	0.186	0.128	0.142	0.159	0.176	0.194	0.214
665	0.220	0.220	0.222	0.186	0.128	0.144	0.159	0.176	0.195	0.215
670	0.220	0.219	0.221	0.185	0.128	0.145	0.158	0.175	0.195	0.214
675	0.221	0.220	0.222	0.186	0.129	0.146	0.158	0.175	0.196	0.214
680	0.220	0.219	0.221	0.186	0.129	0.147	0.156	0.174	0.197	0.214
685	0.221	0.220	0.222	0.186	0.131	0.149	0.156	0.174	0.198	0.215
690	0.221	0.219	0.221	0.186	0.131	0.149	0.155	0.173	0.199	0.214
695	0.222	0.220	0.222	0.187	0.133	0.150	0.155	0.174	0.201	0.215
700	0.223	0.220	0.222	0.187	0.134	0.151	0.154	0.174	0.202	0.216

A illuminants is lower than CIE D65 and EE illuminants. Higher standard deviation of Z tristimulus value is obtained under CIE D65 illuminants.

As reported in Tables 3 and 4, the standard deviation of X tristimulus value for 2-degree standard observer under CIE A illuminant is less than 10-degree standard observer. In a different way, the standard deviation of X tristimulus value for 2-degree standard observer under CIE D65 and EE is more than 10-degree standard observer. The standard deviation of Y tristimulus value for 2-degree standard observer is more than 10-degree standard observer. In addition, the standard deviation of Z tristimulus value for 2-degree standard observer under CIE A, CIE D65 and EE illuminants is more than 10-degree standard observer.

Equation 1 can be expanded into equation 11 for calculating X tristimulus value by using reflectance (R_λ), the relative spectral energy distributions of the illuminant

Table 3: Standard deviation of X, Y and Z tristimulus value of Munsell color samples (2 degree standard observer).

No.	Munsell Hue	X tristimulus			Y tristimulus			Z tristimulus		
		A	D65	EE	A	D65	EE	A	D65	EE
1	R	20.65	16.74	17.82	17.19	16.55	16.68	5.56	17.03	15.62
2	YR	22.11	17.54	18.73	18.57	17.59	17.80	5.25	16.02	14.69
3	Y	23.04	18.14	19.37	20.59	19.80	19.96	5.05	15.26	13.97
4	GY	20.76	17.12	18.12	19.56	19.58	19.56	5.53	16.47	15.05
5	G	16.59	14.72	15.38	16.38	16.95	16.81	5.70	17.17	15.70
6	BG	17.16	15.32	15.99	16.72	17.28	17.14	6.39	19.32	17.67
7	B	17.68	15.64	16.36	16.72	17.12	17.02	6.91	21.09	19.30
8	PB	18.22	15.93	16.71	16.69	16.84	16.80	6.92	21.27	19.48
9	P	19.50	16.78	17.67	17.20	17.09	17.12	6.55	20.14	18.46
10	RP	20.55	17.20	18.21	17.53	17.17	17.25	6.08	18.64	17.09

Table 4: Standard deviation of X, Y and Z tristimulus value of Munsell color samples (10 degree standard observer).

No.	Munsell Hue	X tristimulus			Y tristimulus			Z tristimulus		
		A	D65	EE	A	D65	EE	A	D65	EE
1	R	20.67	16.54	17.63	17.09	16.42	16.54	5.50	16.79	15.63
2	YR	22.17	17.32	18.53	18.36	17.24	17.45	5.18	15.78	14.69
3	Y	23.21	18.00	19.24	20.32	19.23	19.43	4.96	14.97	13.91
4	GY	21.01	17.06	18.09	19.47	19.33	19.34	5.39	16.06	14.88
5	G	16.91	14.77	15.47	16.39	16.93	16.80	5.58	16.80	15.59
6	BG	17.46	15.36	16.08	16.75	17.33	17.19	6.27	18.93	17.57
7	B	17.93	15.65	16.41	16.75	17.20	17.09	6.82	20.73	19.26
8	PB	18.40	15.86	16.69	16.72	16.94	16.89	6.86	20.97	19.50
9	P	19.63	16.64	17.57	17.23	17.16	17.18	6.49	19.87	18.49
10	RP	20.61	17.02	18.06	17.51	17.15	17.22	6.01	18.38	17.11

(E_λ), and the color matching function of standard colorimetric observer $\bar{x}_\lambda$ for 16 wavelengths from 400 to 700 nm with 20 nm intervals is:

$$X = K \times E_{400} \times \bar{x}_{400} \times R_{400} + K \times E_{420} \times \bar{x}_{420} \times R_{420} + ... + K \times E_{700} \times \bar{x}_{700} \times R_{700} \quad (11)$$

If the variance of reflectance at λ is $\sigma^2_{R_\lambda}$, and the variance of X, Y and Z tristimulus values are σ^2_X, σ^2_Y and σ^2_Z, respectively, the variance of X tristimulus value (σ^2_X) can be calculated from variance of reflectance spectra ($\sigma^2_{R_\lambda}$) by using equation 12, which can be obtained by the combination of Equations 6, 8 and 11:

$$\sigma^2_X \approx \left(\frac{\partial X}{\partial R_{400}}\right)^2 \times \sigma^2_{R_{400}} + \left(\frac{\partial X}{\partial R_{420}}\right)^2 \times \sigma^2_{R_{420}} + ... + \left(\frac{\partial X}{\partial R_{700}}\right)^2 \times \sigma^2_{R_{700}} \quad (12)$$

Equation 12 can be simplified into equation 13:

$$\sigma_X^2 \approx \sum_{\lambda=400}^{700} \left(\frac{\partial X}{\partial R_\lambda}\right)^2 \times \sigma_{R_\lambda}^2 \qquad (13)$$

where $\left(\frac{\partial X}{\partial R_\lambda}\right)$ is the partial derivative of X tristimulus value over reflectance at λ, which can be calculated by equation 14:

$$\left(\frac{\partial X}{\partial R_\lambda}\right) = K \times E_\lambda \times \bar{x}_\lambda \qquad (14)$$

Equation 13 can be extended to equation 15 by substitution of $\left(\frac{\partial X}{\partial R_\lambda}\right)$ from equation 14 into equation 12:

$$\sigma_X^2 \approx \sum_{\lambda=400}^{700} \left(K \times E_\lambda \times \bar{x}_\lambda\right)^2 \times \sigma_{R_\lambda}^2 \qquad (15)$$

Similarly, the variance of Y and Z tristimulus values (σ_Y^2 and σ_Z^2) can be obtained from standard deviation of reflectance spectra ($\sigma_{R_\lambda}^2$) by using equations 16 and 17, respectively:

$$\sigma_Y^2 \approx \sum_{\lambda=400}^{700} \left(K \times E_\lambda \times \bar{y}_\lambda\right)^2 \times \sigma_{R_\lambda}^2 \qquad (16)$$

$$\sigma_Z^2 \approx \sum_{\lambda=400}^{700} \left(K \times E_\lambda \times \bar{z}_\lambda\right)^2 \times \sigma_{R_\lambda}^2 \qquad (17)$$

The $\left(\frac{\partial X}{\partial R_\lambda}\right)^2 = \left(K \times E_\lambda \times \bar{x}_\lambda\right)^2$, $\left(\frac{\partial Y}{\partial R_\lambda}\right)^2 = \left(K \times E_\lambda \times \bar{y}_\lambda\right)^2$, $\left(\frac{\partial Z}{\partial R_\lambda}\right)^2 = \left(K \times E_\lambda \times \bar{z}_\lambda\right)^2$, value at 20 nm intervals from 400 to 700 nm are shown in Figures 3, 4, and 5, respectively.

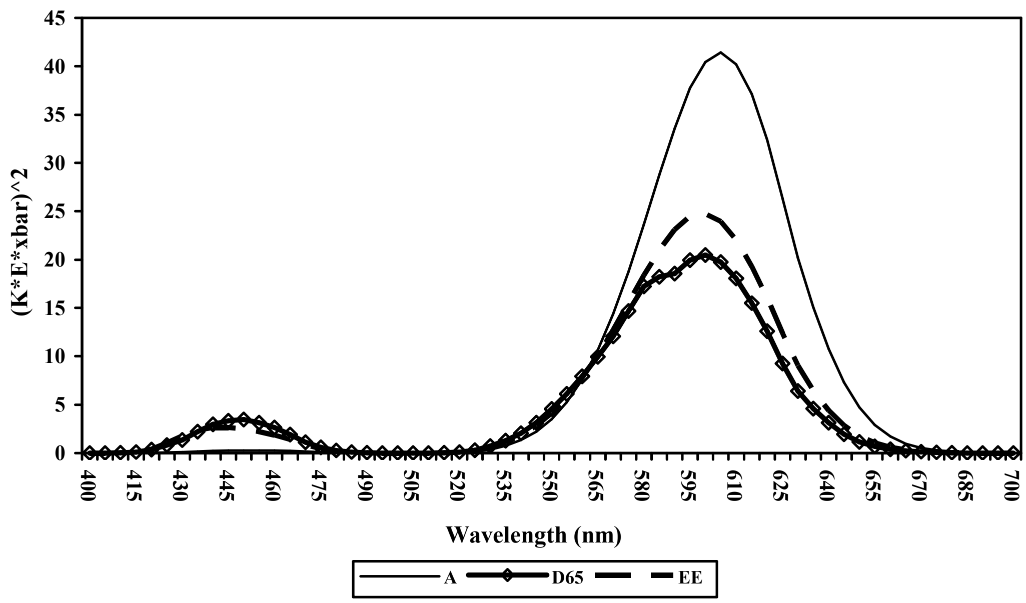

Figure 3: The $\left(\frac{\partial X}{\partial R_\lambda}\right)^2 = \left(K \times E_\lambda \times \bar{x}_\lambda\right)^2$ value under various illuminants **(**CIE (1931) 2-degree standard observer).

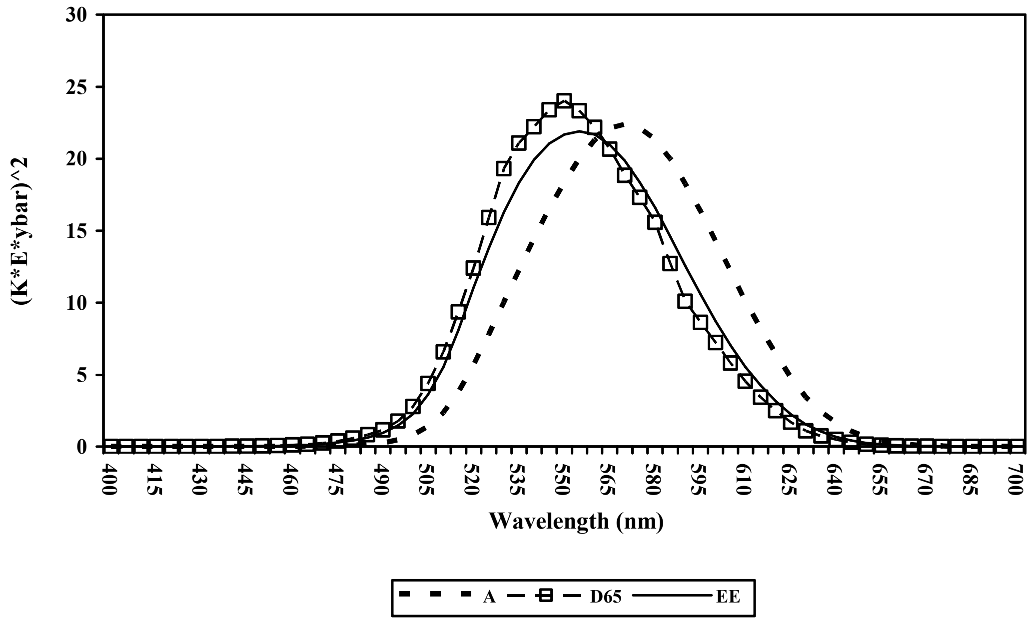

Figure 4. The $\left(\frac{\partial Y}{\partial R_\lambda}\right)^2 = \left(K \times E_\lambda \times \bar{y}_\lambda\right)^2$ value under various illuminants **(**CIE (1931) 2-degree standard observer).

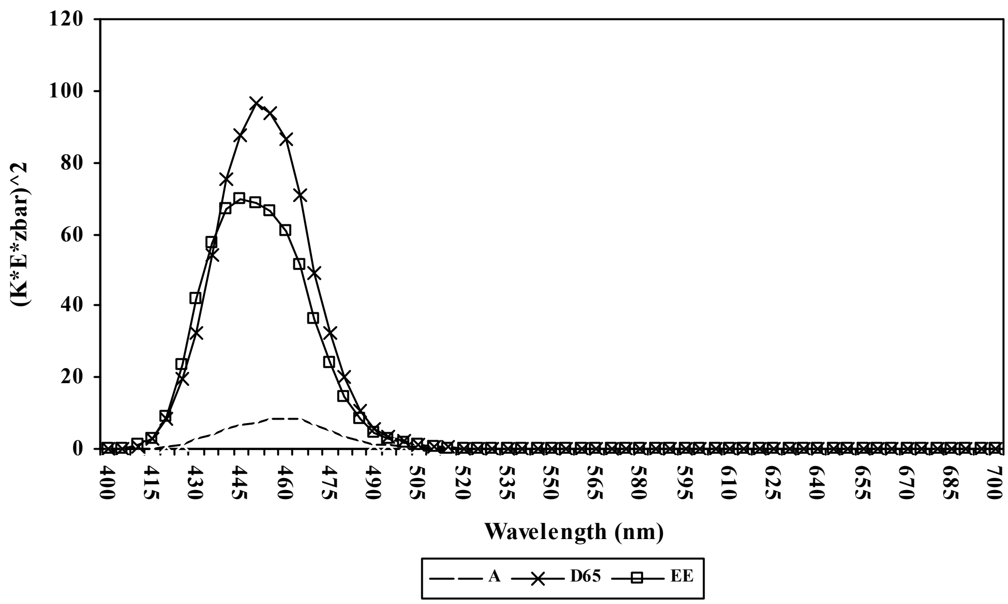

Figure 5: The $\left(\frac{\partial Z}{\partial R_\lambda}\right)^2 = \left(K \times E_\lambda \times \bar{z}_\lambda\right)^2$ value under various illuminants **(**CIE (1931) 2-degree standard observer)..

As shown in these figures, the transportation of the error and the variance from reflectance data into CIE tristimulus values depends on relative spectral power distribution of illuminants and the color matching function of standard colorimetric observers. For X tristimulus value, the $\left(\frac{\partial X}{\partial R_\lambda}\right)^2 = \left(K \times E_\lambda \times \bar{x}_\lambda\right)^2$ values under CIE A illuminant are more than CIE D65 and EE illuminants. For Y tristimulus value, the

$\left(\frac{\partial Y}{\partial R_\lambda}\right)^2 = (K \times E_\lambda \times \bar{y}_\lambda)^2$ values are equal for CIE A, CIE D65 and EE illuminants. For Z tristimulus value, the $\left(\frac{\partial Z}{\partial R_\lambda}\right)^2 = (K \times E_\lambda \times \bar{z}_\lambda)^2$ value under CIE D65 illuminant is more than CIE, A and EE illuminants. Equations 11, 12 and 13 show that the standard deviation and variance of CIE tristimulus values increase with increasing their respective coefficients and weights. This implies that in a weighted sum of variables, the variable with the largest weight will have the largest weight in the variance of the total.

The uncertainty propagation from reflectance spectra to colorimetric data depends on sample reflectance spectra form and spectral power distribution (SPD) of illuminants and color matching function of standard observers.

In the next step, the standard deviation of X, Y and Z tristimulus values under various conditions was estimated by using standard deviation of reflectance spectra at each wavelength according to equations 15, 16 and 17, respectively.
The relative error, shown in equation 18 for X, Y and Z tristimulus values are listed in Table 5.

$$Error\% = 100 \times \frac{\left|STD_{actual} - STD_{predicted}\right|}{STD_{actual}} \qquad (18)$$

where Error% is relative error, STD_{actual} and $STD_{predioted}$ are actual and predicted standard deviations obtained by equations 15, 16 and 17, respectively.

As shown in this table, the accuracy of the standard deviation estimation for Z tristimulus value is higher than those for X and Y tristimulus values.

3.1. Applications

Table 5: Evaluation of standard deviation of X, Y and Z tristimulus from standard deviation of reflectance spectra.

Tristimulus	Illuminant	Standard observer	Mean	Max	Min	STD
X	CIE A	2	2.562	7.057	0.981	1.975
		10	2.657	7.332	1.005	2.030
	CIE D65	2	3.996	8.695	1.612	2.237
		10	4.026	8.677	1.616	2.245
	EE	2	4.026	8.677	1.616	2.245
		10	3.861	8.606	1.540	2.192
Y	CIE A	2	3.108	8.492	0.950	2.259
		10	3.309	8.579	1.172	2.215
	CIE D65	2	3.172	7.462	1.330	1.929
		10	3.482	7.376	1.451	1.940
	EE	2	3.219	7.842	1.279	2.017
		10	3.524	7.807	1.567	2.003
Z	CIE A	2	1.139	3.412	0.115	1.132
		10	0.926	2.592	0.064	0.887
	CIE D65	2	0.820	2.232	0.073	0.754
		10	0.686	1.724	0.065	0.587
	EE	2	0.839	2.268	0.081	0.761
		10	0.696	1.731	0.072	0.585

4. Conclusions

This study describes the relationship between colorimetric and spectrophotometric errors. The obtained results indicate that the error propagation from reflectance spectra to CIE tristimulus values depends on both illuminants SPD's and standard observers. The effect of the illuminants SPD's is more than standard colorimetric observer. X tristimulus value under CIE A illuminant, Y tristimulus value under CIE A illuminant and Z tristimulus value under CIE D65 illuminant showed the most standard deviations in their corresponding groups. The standard deviation of X and Y tristimulus values under 2-degree standard colorimetric observers is more than 10-degree standard colorimetric observers. The standard deviation of Z tristimulus value under 2-degree standard colorimetric observer is less than 10-degree standard colorimetric observers. The effect of spectral power distribution of illuminants and color matching function of standard colorimetric observer combination depends on their respective coefficients and weights. The proposed relationship was used to estimate the variance of CIE tristimulus values under various illuminants from variance of reflectance spectra. The proposed formulae give reasonably small estimated errors.

5. References

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